Law of Sines: Start with *sin (A)* and *sin(C)*.

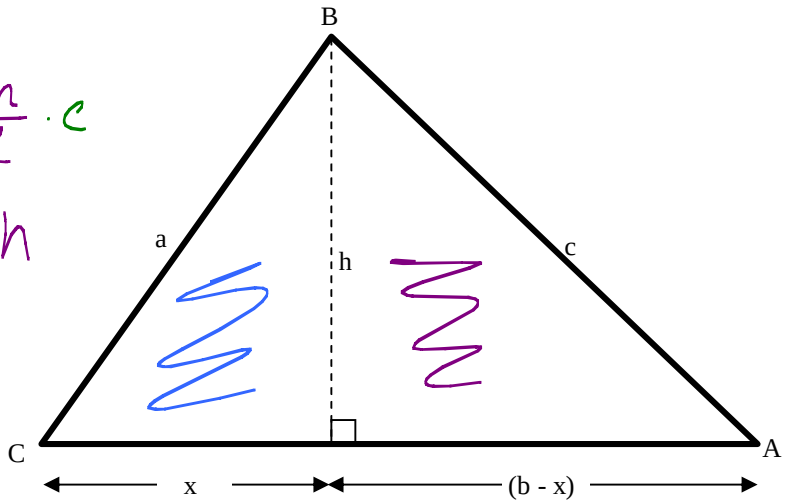
PROOF :

$a \cdot \sin C = \frac{h}{a} \cdot a$ | $c \cdot \sin A = \frac{h}{c} \cdot c$

$a \cdot \sin C = h$ | $c \cdot \sin A = h$

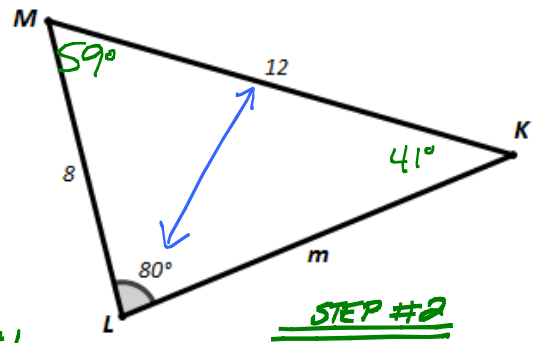
$\frac{a \cdot \sin C}{ac} = \frac{c \sin A}{ac}$

$\frac{\sin C}{c} = \frac{\sin A}{a}$



$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

1. Find the unknown sides and angles of each triangle using the **Law of Sines.**



STEP #1
(LAW OF SINES)

$\frac{\sin 80}{12} \times \frac{\sin(K)}{8}$

$\frac{12 \cdot \sin(K)}{12} = \frac{8 \sin 80}{12}$

$\sin(K) \approx 0.6565$

$\sin^{-1}(\sin(K)) \approx \sin^{-1}(0.6565)$

$K \approx 41°$

```
8*sin(80)/12
          .656538502
sin⁻¹(Ans)
          41.03641053
```

STEP #2
(INTERIOR ANGLES OF A TRIANGLE SUM TO 180°)

$M = 180 - 80 - 41$

$M = 59°$

STEP #3
(LAW OF SINES)

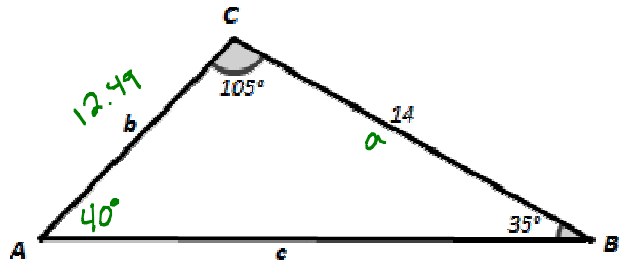
$\frac{\sin 80}{12} \times \frac{\sin 59}{M}$

$\frac{12 \sin 59}{\sin 80} = \frac{M \cdot \sin 80}{\sin 80}$

$10.44 \approx M$

```
12*sin(59)/sin(8
0)
          10.44468586
```

m	10.44
m∠M	59°
m∠K	41



STEP #1
(INTERIOR ANGLES OF A TRIANGLE SUM TO 180°)

$m\angle A = 180 - 105 - 35$

$m\angle A = 40°$

STEP #2
(LAW OF SINES)

$\frac{\sin 40}{14} \times \frac{\sin 35}{b}$

$\frac{14 \cdot \sin 35}{\sin 40} = \frac{b \cdot \sin 40}{\sin 40}$

$12.49 \approx b$

STEP # 3
(LAW OF SINES)

$\frac{\sin 40}{14} \times \frac{\sin 105}{c}$

$\frac{14 \sin 105}{\sin 40} = \frac{c \cdot \sin 40}{\sin 40}$

$21.04 \approx c$

c	21.04
b	12.49
m∠A	40

2. A student was trying to determine the height of the Washington monument from a distance. So, he measured two angles of elevation 44 meters apart. The angle of elevation the furthest away from the monument measured to be 25° and the closest angle of elevation measured 28°. The student determining the angles is 1.6 Meters tall from his feet to his eyeballs. Find the Height = ≈ 168.4 METERS Distance away = ≈ 313.7 METERS

STEP #1
(LINEAR PAIRS SUM TO 180°)
$m\angle ACB = 180 - 28 = 152°$

STEP #2
(INTERIOR ANGLES OF A TRIANGLE SUM TO 180°)
$m\angle ABC = 180 - 25 - 152 = 3°$

STEP #3
(LAW OF SINES)
$\frac{\sin 3}{44} \times \frac{\sin 25}{a}$

$\frac{44\sin 25}{\sin 3} = \frac{a \cdot \sin 3}{\sin 3}$

$355.3 \approx a$

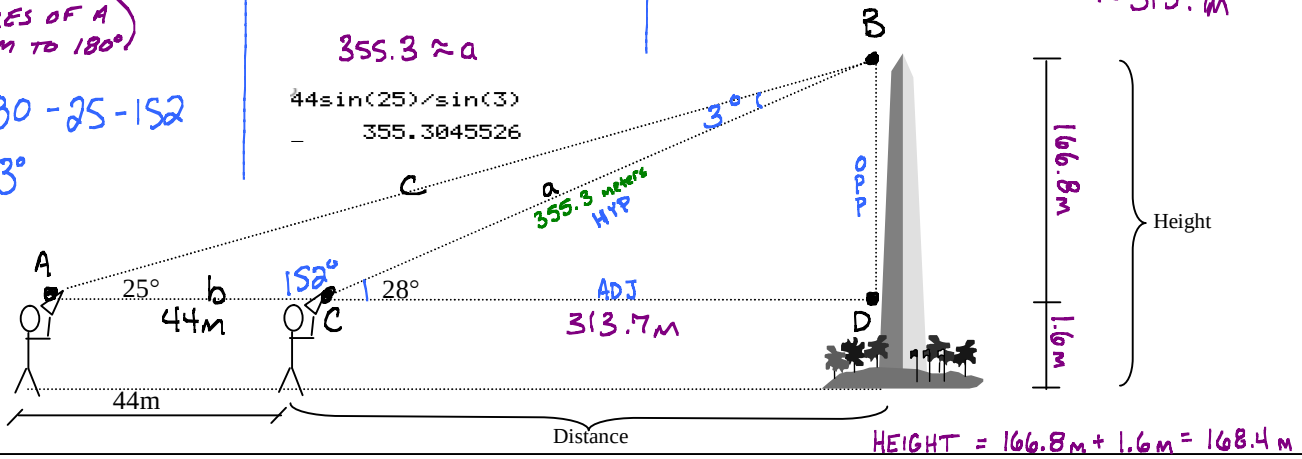
44sin(25)/sin(3)
355.3045526

STEP #4
(BASIC TRIG)
$\frac{\sin 28}{1} \times \frac{x}{355.3}$

$x = 355.3 \sin 28$
$x \approx 166.8$ m

STEP #5
(BASIC TRIG)
$\frac{\cos 28}{1} \times \frac{A}{355.3}$

$A = 355.3 \cos 28$
≈ 313.7m



HEIGHT = 166.8m + 1.6m = 168.4 m

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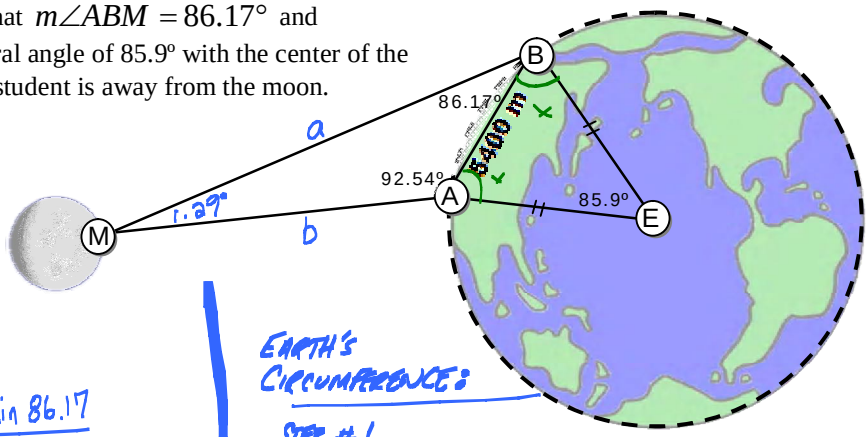
8. Two students that are on the same longitudinal line are approximately 5400 miles apart. The used an inclinometer, a little geometry, and a tangent line to determine the that $m\angle ABM = 86.17°$ and $m\angle BAM = 92.54°$. The two students form a central angle of 85.9º with the center of the earth. Given this information determine how far each student is away from the moon.



DISTANCE TO THE MOON

STEP #1
(TRIANGLE'S INTERIOR ANGLES SUM TO 180°)
$m\angle M = 180 - 92.54 - 86.17 = 1.29°$

STEP # 2
$\frac{\sin 1.29}{5400} \times \frac{\sin(92.54)}{a}$

$\frac{5400\sin(92.54)}{\sin(1.29)} = \frac{a \cdot \sin(1.29)}{\sin(1.29)}$

$239627.4\text{m} \approx a$

5400*sin(92.54)/
sin(1.29)
239627.4032

a: MB ≈ 239627 miles

STEP #3
$\frac{\sin 1.29}{5400} \times \frac{\sin 86.17}{b}$

$\frac{5400\sin 86.17}{\sin(1.29)} = \frac{b\sin 1.29}{\sin(1.29)}$

$239327 \text{ miles} \approx b$

5400*sin(86.17)/
sin(1.29)
239327.3596

b: MA ≈ 239327 miles

EARTH'S CIRCUMFERENCE:

STEP #1
$m\angle BAE = \frac{180 - 85.9}{2}$

$m\angle BAE = 47.05°$

STEP #2
$\frac{\sin 85.9}{5400} \times \frac{\sin 47.05}{a}$

$\frac{5400\sin 47.05}{\sin 85.9} = \frac{a \cdot \sin 85.9}{\sin 85.9}$

$3963 \text{ miles} \approx a = \text{RADIUS}$

STEP #3
$C = 2\pi r$
$C = 2\pi(3963)$
$C \approx 24{,}900$ miles

Use this information to find the radius of the Earth and then the circumference ($C = 2\pi r$). $\approx$ 24,900 MILES